

# Experimentally Deriving Newton’s Universal Law of Gravitation by Measuring Mass, Distance, and G in a Simulation

**Name:** Keshav Anand

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## OBJECTIVES

- Use the Gravitational force simulation to determine the dependence of the gravitational force on the mass of the objects involved.
- Use the same simulation to determine the dependence of the gravitational force on the distance between the two masses.
- Determine the experimental value of the universal gravitational constant ( $G$ ). (This is what relates the gravitational force to the masses and distance rather than being these proportional.  $G$  must be included in your final equation.)
- Determine an Equation for the Universal Law of Gravitation based on your data, using only symbols.

## INTRODUCTION

### BACKGROUND

The concept of gravitational force has been misunderstood for most of history, commonly attributed to divine or supernatural causes until Sir Isaac Newton published his groundbreaking *Principia*. In the *Principia*, Newton asserted that every mass exerts an attractive force on every other mass, a phenomenon described by Newton’s Universal Law of Gravitation (NLUG). This law states that the magnitude of the gravitational force between two masses is

$$F_g = G \frac{m_1 m_2}{r^2} \quad (1)$$

where  $F$  is the gravitational force,  $m_1$  and  $m_2$  are the interacting masses,  $r$  is the distance between their centers.  $G$  is the universal gravitational constant, a constant of proportionality that has been calculated to be

$$G = 6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}.$$

In the scientific community, NLUG is treated as an absolute truth, and many important discoveries and applications rely on its accuracy. From engineering to astrophysics, NLUG has profound importance, and its validity is vital to the functioning of scientific advancement.

## PURPOSE

This lab aims to use a computer simulation to verify NLUG by deriving the relationship between gravitational force, masses of objects, and the distance between them. First, a gravitation simulation will be used to derive the relationship between objects' masses and the gravitational force. Then, the same process will be repeated with comparing gravitational force to the distance between objects. Data collected from these two setups will be used to (hopefully) re-establish the relationship of proportionality proposed by Newton, and the collected data will also solve for the universal Gravitational constant,  $G$ .

## RESEARCH PROBLEM

The research objective for this project is to verify the gravitational relationship between two objects and verify the constant  $G$ . The primary problem of investigation is that theoretical mathematics often fails to adequately capture a true relationship in the real world. Moreover, using physical objects and tools of measurements can result in unwanted noisy data and is limited by the precision of measurement. Ergo, a simulation bridges this gap, allowing for an accurate verification of NLUG.

## METHODOLOGY

### MATERIALS AND RESOURCES

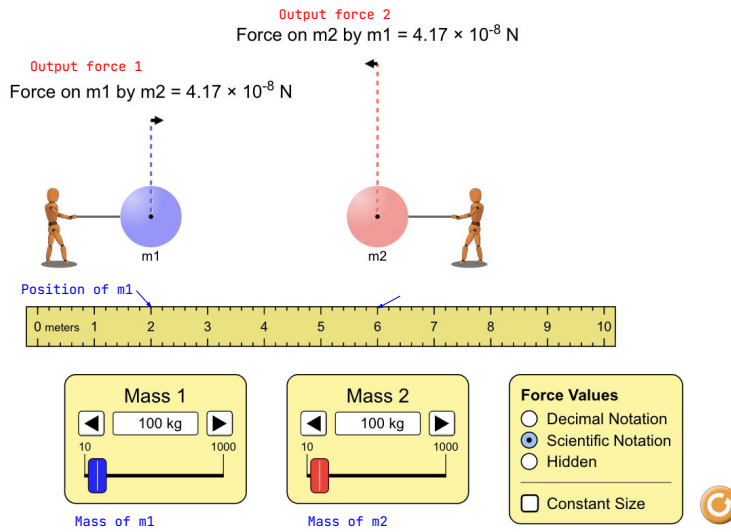
As this lab was performed within a simulation, all physical materials are limited to a computer with at least 400 MB of memory to render the simulation.

Within the simulation, the simulated materials include

- Adjustable Mass,  $m_1$
- Adjustable Mass,  $m_2$
- 10 Meter Scale
- Automatic Force Scale to Measure Gravitational Attraction
- Two simulated people holding  $m_1$  and  $m_2$  from colliding into each other due to gravitation

### EXPERIMENTAL SETUP

Note that in Figure 1, all inputs (independent variables) are denoted in blue, whereas outputs (dependent variables) are denoted in red



**Figure 1:** Experimental setup for the gravity simulation

## PROCEDURE

1. Set the location of mass 1 to exactly 2 meters on the scale, and set mass 2 to exactly 6 meters on the scale, with a distance between of 4 meters
2. Set the mass of objects 1 and 2 to exactly 100 kg
3. Set the force values to scientific notation, and uncheck the option for masses of constant size
4. Leaving the mass of  $m_2$  constant, change the mass of  $m_1$  to be the values listed below, and record both the force on  $m_1$  by  $m_2$  and the force on  $m_2$  by  $m_1$

Mass values for  $m_1$  (kg): 50, 100, 250, 500, 750, 1000

5. Reset the simulation as detailed by steps 1-3
6. Leaving the mass of  $m_1$  constant, change the mass of  $m_2$  to be the values listed below, and record both the force on  $m_1$  by  $m_2$  and the force on  $m_2$  by  $m_1$

Mass values for  $m_2$  (kg): 50, 100, 250, 500, 750, 1000

7. Reset the simulation as detailed by steps 1-3
8. Change the masses of both  $m_1$  and  $m_2$  to be the values listed below, and record both the force on  $m_1$  by  $m_2$  and the force on  $m_2$  by  $m_1$

Mass values for  $m_1$  and  $m_2$  (kg): 50, 100, 250, 500, 750, 1000

9. Reset the simulation as detailed by steps 1-3

10. Leave  $m_2$  at 10 meters on the scale (align the black dot for center of mass), and move  $m_1$  based on its center to the below values on the scale, and record both the force on  $m_1$  by  $m_2$  and the force on  $m_2$  by  $m_1$

Position values for  $m_1$  (m): 0, 2, 4, 6, 8

11. Reset the simulation as detailed by steps 1-3

12. Leave  $m_1$  at 0 meters on the scale (align the black dot for center of mass), and move  $m_2$  based on its center to the below values on the scale, and record both the force on  $m_1$  by  $m_2$  and the force on  $m_2$  by  $m_1$

Position values for  $m_1$  (m): 10, 8, 6, 4, 2

Note that the above steps require the following raw data the be collected at each datapoint

- Position of the center of mass of  $m_1$ , (m)
- Position of the center of mass of  $m_2$ , (m)
- Mass of  $m_1$ , (kg)
- Mass of  $m_2$ , (kg)
- Force on  $m_1$  by  $m_2$ , (N)
- Force on  $m_2$  by  $m_1$ , (N)

## RESULTS

### RAW DATA

While all data was collected jointly, the five separate experimental setups can be split up into the following tables for convenience:

**Table 1:** Force between two masses while varying  $m_1$ .

<b>Trial</b>	$m_1$ (kg)	$m_2$ (kg)	$x_1$ (m)	$x_2$ (m)	$F_{1 \rightarrow 2}$ (N)	$F_{2 \rightarrow 1}$ (N)
1	50	100	2.00	6.00	$2.09 \times 10^{-8}$	$2.09 \times 10^{-8}$
2	100	100	2.00	6.00	$4.17 \times 10^{-8}$	$4.17 \times 10^{-8}$
3	250	100	2.00	6.00	$1.04 \times 10^{-7}$	$1.04 \times 10^{-7}$
4	500	100	2.00	6.00	$2.09 \times 10^{-7}$	$2.09 \times 10^{-7}$
5	750	100	2.00	6.00	$3.13 \times 10^{-7}$	$3.13 \times 10^{-7}$
6	1000	100	2.00	6.00	$4.17 \times 10^{-7}$	$4.17 \times 10^{-7}$

**Table 2:** Force between two masses while varying  $m_2$ .

<b>Trial</b>	$m_1$ (kg)	$m_2$ (kg)	$x_1$ (m)	$x_2$ (m)	$F_{1\rightarrow 2}$ (N)	$F_{2\rightarrow 1}$ (N)
1	100	50	2.00	6.00	$2.09\times 10^{-8}$	$2.09\times 10^{-8}$
2	100	100	2.00	6.00	$4.17\times 10^{-8}$	$4.17\times 10^{-8}$
3	100	250	2.00	6.00	$1.04\times 10^{-7}$	$1.04\times 10^{-7}$
4	100	500	2.00	6.00	$2.09\times 10^{-7}$	$2.09\times 10^{-7}$
5	100	750	2.00	6.00	$3.13\times 10^{-7}$	$3.13\times 10^{-7}$
6	100	1000	2.00	6.00	$4.17\times 10^{-7}$	$4.17\times 10^{-7}$

**Table 3:** Force between equal masses while varying  $m_1 = m_2$ .

<b>Trial</b>	$m_1$ (kg)	$m_2$ (kg)	$x_1$ (m)	$x_2$ (m)	$F_{1\rightarrow 2}$ (N)	$F_{2\rightarrow 1}$ (N)
1	50	50	2.00	6.00	$1.04\times 10^{-8}$	$1.04\times 10^{-8}$
2	100	100	2.00	6.00	$4.17\times 10^{-8}$	$4.17\times 10^{-8}$
3	250	250	2.00	6.00	$2.61\times 10^{-7}$	$2.61\times 10^{-7}$
4	500	500	2.00	6.00	$1.04\times 10^{-6}$	$1.04\times 10^{-6}$
5	750	750	2.00	6.00	$2.35\times 10^{-6}$	$2.35\times 10^{-6}$
6	1000	1000	2.00	6.00	$4.17\times 10^{-6}$	$4.17\times 10^{-6}$

**Table 4:** Force between two masses while varying  $x_1$ .

<b>Trial</b>	$m_1$ (kg)	$m_2$ (kg)	$x_1$ (m)	$x_2$ (m)	$F_{1\rightarrow 2}$ (N)	$F_{2\rightarrow 1}$ (N)
1	100	100	0.00	10.00	$6.67\times 10^{-9}$	$6.67\times 10^{-9}$
2	100	100	2.00	10.00	$1.04\times 10^{-8}$	$1.04\times 10^{-8}$
3	100	100	4.00	10.00	$1.85\times 10^{-8}$	$1.85\times 10^{-8}$
4	100	100	6.00	10.00	$4.17\times 10^{-8}$	$4.17\times 10^{-8}$
5	100	100	8.00	10.00	$1.67\times 10^{-7}$	$1.67\times 10^{-7}$

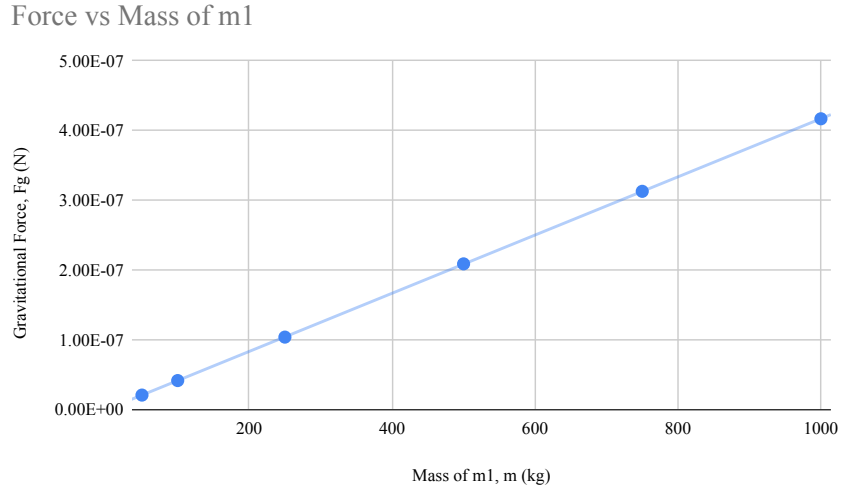
**Table 5:** Force between two masses while varying  $x_2$ .

Trial	$m_1$ (kg)	$m_2$ (kg)	$x_1$ (m)	$x_2$ (m)	$F_{1\rightarrow 2}$ (N)	$F_{2\rightarrow 1}$ (N)
1	100	100	0.00	10.00	$6.67\times 10^{-9}$	$6.67\times 10^{-9}$
2	100	100	0.00	8.00	$1.04\times 10^{-8}$	$1.04\times 10^{-8}$
3	100	100	0.00	6.00	$1.85\times 10^{-8}$	$1.85\times 10^{-8}$
4	100	100	0.00	4.00	$4.17\times 10^{-8}$	$4.17\times 10^{-8}$
5	100	100	0.00	2.00	$1.67\times 10^{-7}$	$1.67\times 10^{-7}$

## DISCUSSION AND ANALYSIS

The first major observation that can be made using the raw data is that for all data points (in all 5 tables),  $F_{1\rightarrow 2}$  (N) =  $F_{2\rightarrow 1}$  (N). This observation is **Newton's Third Law**, as the force of one mass on another is equal to the other mass on it. Hence, these two forces can be replaced by one force column, denoted as  $F_g$ , which is the force of gravitational attraction between the two objects. Also notes that the direction of these two forces are always towards each others, which is why gravitation is known as a force of attraction.

Then, look to Table 1. Observe that there is a relationship between the changing variable ( $m_1$ ) and the gravitational force. As the other variables are all constant in this experiment, the relationship between  $m_1$  and  $F_g$  can be graphed as follows.



**Figure 2:**  $F_g$  vs  $m_1$  graphed

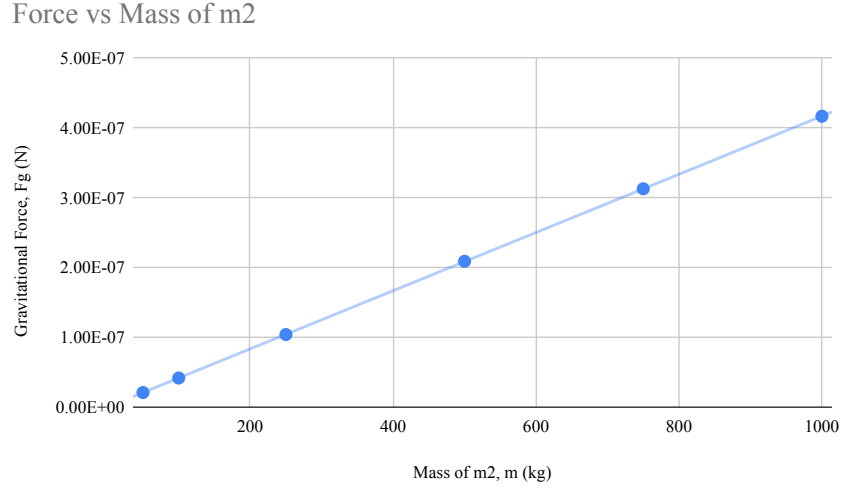
Observing Figure 2, there is a linear trend between the gravitational force and the mass of object 1. This can be represented by the proportionality:

$$F_g \propto m_1.$$

Hence, the relationship can be described with the below equation, where  $k_1$  is simply a constant:

$$F_g = k_1 \times m_1 \quad (2)$$

Applying the same logic to Table 2 yields the following relationship between  $F_g$  and  $m_2$



**Figure 3:**  $F_g$  vs  $m_2$  graphed

Note that again, there is a linear trend, implying

$$F_g \propto m_2.$$

Hence, the relationship can be described with the below equation, where  $k_2$  is simply another constant:

$$F_g = k_2 \times m_2. \quad (3)$$

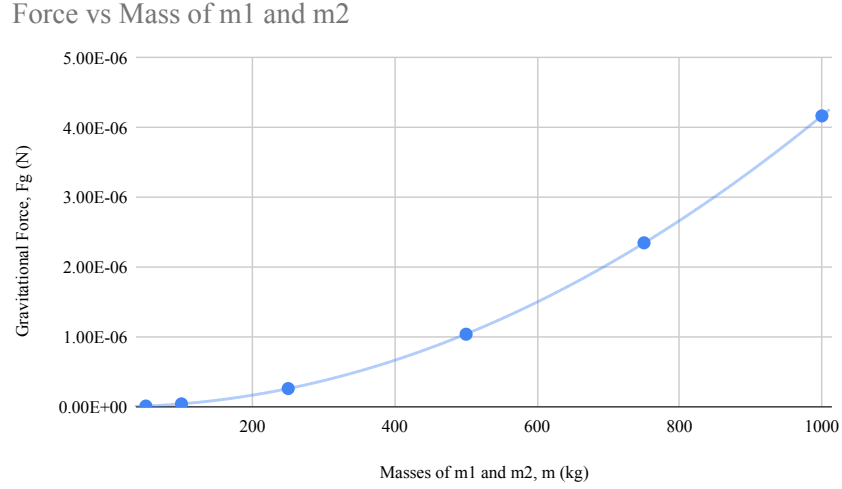
Looking at equation 2 and 3, they can be safely combined into one equation, where there is yet another constant of proportionality  $k_3$ :

$$F_g = k_3(m_1 \times m_2). \quad (4)$$

Note, that when  $m_1$  and  $m_2$  are the same, equation 4 simplifies to

$$F_g = k_3 \times m^2.$$

Looking at Table 3, this derived relationship can be verified by graphing as follows:



**Figure 4:**  $F_g$  vs mass of  $m_1 = m_2$

As suggested by equation 4, there is a proportional quadratic relationship between the masses of the objects and the resulting gravitational force (see Figure 4). Therefore, equation 4 is validated by the simulation.

Moving onto Tables 4 and 5, the output columns for the gravitational force appear identical. Upon further inspection, the gravitational force seems to exhibit a dependency on the distance between the masses. A processed data table can be made combining Tables 4 and 5, showcasing the relationship between  $F_g$  and  $r := |x_1 - x_2|$ .

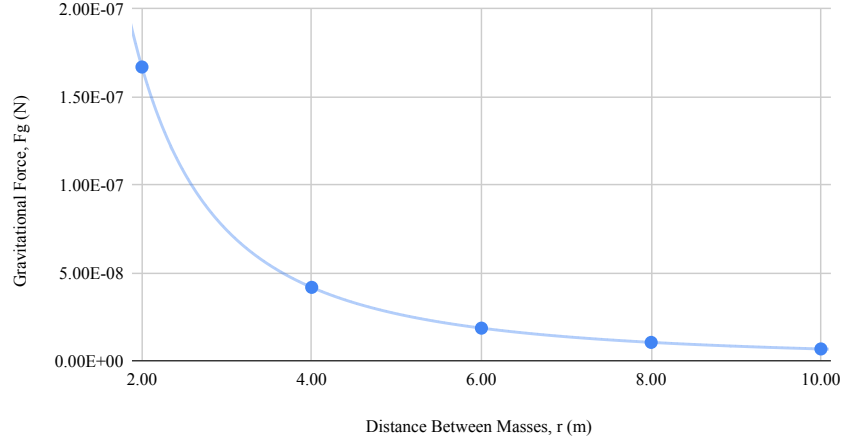
**Table 6:** Gravitational force versus distance between two masses.

Distance $r$ (m)	Gravitational Force $F_g$ (N)
2.00	$1.67 \times 10^{-7}$
4.00	$4.17 \times 10^{-8}$
6.00	$1.85 \times 10^{-8}$
8.00	$1.04 \times 10^{-8}$
10.00	$6.67 \times 10^{-9}$

This new relationship is graphed in the figure below:



Force vs Distance Between Masses



**Figure 5:**  $F_g$  vs  $r = x_2 - x_1$

The shown trendline suggests a proportional fit to  $\frac{1}{r^2}$  as follows:

$$F_g \propto \frac{1}{r^2},$$

so

$$F_g = k_4 \times \frac{1}{r^2}. \quad (5)$$

When combined with equation 4, NLUG pops out, and the proportionality constant can be denoted as  $G$ :

$$F_g = G \frac{m_1 m_2}{r^2} \quad (1)$$

The final step is to solve for the proportionality constant,  $G$ . As the relationship has been proven, any data point can be used to solve for this constant. For simplicity, the control data point (used for setup) will be used, where  $m_1 = 100$  kg,  $m_2 = 100$  kg,  $r = x_2 - x_1 = 6 \text{ m} - 2 \text{ m} = 4 \text{ m}$ . We have

$$F_g = G \left( \frac{m_1 m_2}{r^2} \right) = G \left( \frac{100 \text{ kg} \times 100 \text{ kg}}{4^2 \text{ m}^2} \right) = 4.17 \times 10^{-8} \text{ N}$$

$$G = \frac{F_g r^2}{m_1 m_2} = \frac{(4.17 \times 10^{-8} \text{ N})(4 \text{ m})^2}{100 \text{ kg} \times 100 \text{ kg}} = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

Ergo, the data from the simulation can be used to derive NLUG and solve for  $G$ , the universal gravitational constant:

$$F_g = G \frac{m_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \quad (6)$$

the force acting on the block can be calculated using Newton's Second Law, which can be used to calculate the frictional coefficients.

# CONCLUSION

## ERROR ANALYSIS

One major source of error within this lab is the assumption that the simulation is entirely functional in simulating gravitational attraction. In an real-world scenario, there are other external forces (friction, air resistance, electromagnetic forces) that can impact the gravitational attraction, and if the simulation doesn't properly isolate the gravitational force, the final relationship and the calculated gravitational constants can be off. Additionally, the small nature of  $G$  means that minor errors in any input variables or rounding errors have a profound impact on the calculated force and gravitational constant. Moreover, the simulation only offers discrete input increments for distance and mass, whereas the scale (particularly for distance) is free to move at any point in the screen. Hence, the distance intervals may be slightly inaccurate as they depend on the accurate placing of the two objects based on the scale. Offering continuous distance inputs would offer a slight improvement. Due to the relatively high number of trials conducted, these errors are relatively negligible in finding proportionalities; however, they can significantly impact the calculation of  $G$ . Hence,  $G$  was calculated using the default values for distance, ensuring that there would be no inaccuracy of the data point.