Experimentally Deriving Newton's Universal Law of Gravitation by Measuring Mass, Distance, and G in a Simulation

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Objectives

- Use the Gravitational force simulation to determine the dependence of the gravitational force on the mass of the objects involved.
- Use the same simulation to determine the dependence of the gravitational force on the distance between the two masses.
- Determine the experimental value of the universal gravitational constant (G). (This is what relates the gravitational force to the masses and distance rather than being these proportional. G must be included in your final equation.)
- Determine an Equation for the Universal Law of Gravitation based on your data, using only symbols.

Introduction

Background

The concept of gravitational force has been misunderstood for most of history, commonly attributed to divine or supernatural causes until Sir Isaac Newton published his groundbreaking *Principia*. In the Principia, Newton asserted that every mass exerts an attractive force on every other mass, a phenomenon described by Newton's Universal Law of Gravitation (NLUG). This law states that the magnitude of the gravitational force between two masses is

$$F_g = G\frac{m_1 m_2}{r^2} \tag{1}$$

where F is the gravitational force, m_1 and m_2 are the interacting masses, r is the distance between their centers. G is the universal gravitational constant, a constant of proportionality that has been calculated to be

$$G = 6.674 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}.$$

In the scientific community, NLUG is treated as an absolute truth, and many important discoveries and applications rely on its accuracy. From engineering to astrophysics, NLUG has profound importance, and its validity is vital to the functioning of scientific advancement.

Purpose

This lab aims to use a computer simulation to verify NLUG by deriving the relationship between gravitational force, masses of objects, and the distance between them. First, a gravitation simulation will be used to derive the relationship between objects' masses and the gravitational force. Then, the same process will be repeated with comparing gravitational force to the distance between objects. Data collected from these two setups will be used to (hopefully) re-establish the relationship of proportionality proposed by Newton, and the collected data will also solve for the universal Gravitational constant, G.

Research Problem

The research objective for this project is to verify the gravitational relationship between two objects and verify the constant G. The primary problem of investigation is that theoretical mathematics often fails to adequately capture a true relationship in the real world. Moreover, using physical objects and tools of measurements can result in unwanted noisy data and is limited by the precision of measurement. Ergo, a simulation bridges this gap, allowing for an accurate verification of NLUG.

Methodology

Materials and Resources

As this lab was performed within a simulation, all physical materials are limited to a computer with at least 400 MB of memory to render the simulation.
Within the simulation, the simulated materials include

- Adjustable Mass, m_1
- Adjustable Mass, m_2
- 10 Meter Scale
- Automatic Force Scale to Measure Gravitational Attraction
- Two simulated people holding m_1 and m_2 from colliding into each other due to gravitation

Experimental Setup

Note that in Figure 1, all inputs (independent variables) are denoted in blue, whereas outputs (dependent variables) are denoted in red

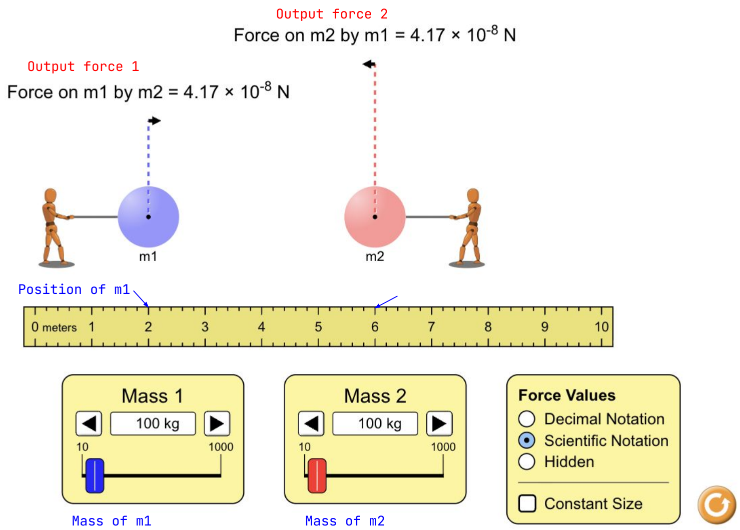

Figure 1: Experimental setup for the gravity simulation

Procedure

1. Set the location of mass 1 to exactly 2 meters on the scale, and set mass 2 to exactly 6 meters on the scale, with a distance between of 4 meters
2. Set the mass of objects 1 and 2 to exactly 100 kg
3. Set the force values to scientific notation, and uncheck the option for masses of constant size
4. Leaving the mass of m_2 constant, change the mass of m_1 to be the values listed below, and record both the force on m_1 by m_2 and the force on m_2 by m_1

   Mass values for m_1 (kg): 50, 100, 250, 500, 750, 1000
5. Reset the simulation as detailed by steps 1-3
6. Leaving the mass of m_1 constant, change the mass of m_2 to be the values listed below, and record both the force on m_1 by m_2 and the force on m_2 by m_1

   Mass values for m_2 (kg): 50, 100, 250, 500, 750, 1000
7. Reset the simulation as detailed by steps 1-3
8. Change the masses of both m_1 and m_2 to be the values listed below, and record both the force on m_1 by m_2 and the force on m_2 by m_1

   Mass values for m_1 and m_2 (kg): 50, 100, 250, 500, 750, 1000
9. Reset the simulation as detailed by steps 1-3

10. Leave m_2 at 10 meters on the scale (align the black dot for center of mass), and move m_1 based on its center to the below values on the scale, and record both the force on m_1 by m_2 and the force on m_2 by m_1

    Position values for m_1 (m): 0, 2, 4, 6, 8

11. Reset the simulation as detailed by steps 1-3

12. Leave m_1 at 0 meters on the scale (align the black dot for center of mass), and move m_2 based on its center to the below values on the scale, and record both the force on m_1 by m_2 and the force on m_2 by m_1

    Position values for m_1 (m): 10, 8, 6, 4, 2

Note that the above steps require the following raw data the be collected at each datapoint

- Position of the center of mass of m_1, (m)
- Position of the center of mass of m_2, (m)
- Mass of m_1, (kg)
- Mass of m_2, (kg)
- Force on m_1 by m_2, (N)
- Force on m_2 by m_1, (N)

Results

Raw Data

While all data was collected jointly, the five separate experimental setups can be split up into the following tables for convenience:

Table 1: Force between two masses while varying m_1.

Trial	m_1 **(kg)**	m_2 **(kg)**	x_1 **(m)**	x_2 **(m)**	$F_{1\to2}$ **(N)**	$F_{2\to1}$ **(N)**
1	50	100	2.00	6.00	2.09×10^{-8}	2.09×10^{-8}
2	100	100	2.00	6.00	4.17×10^{-8}	4.17×10^{-8}
3	250	100	2.00	6.00	1.04×10^{-7}	1.04×10^{-7}
4	500	100	2.00	6.00	2.09×10^{-7}	2.09×10^{-7}
5	750	100	2.00	6.00	3.13×10^{-7}	3.13×10^{-7}
6	1000	100	2.00	6.00	4.17×10^{-7}	4.17×10^{-7}

Table 2: Force between two masses while varying m_2.

Trial	m_1 **(kg)**	m_2 **(kg)**	x_1 **(m)**	x_2 **(m)**	$F_{1\to2}$ **(N)**	$F_{2\to1}$ **(N)**
1	100	50	2.00	6.00	2.09×10^{-8}	2.09×10^{-8}
2	100	100	2.00	6.00	4.17×10^{-8}	4.17×10^{-8}
3	100	250	2.00	6.00	1.04×10^{-7}	1.04×10^{-7}
4	100	500	2.00	6.00	2.09×10^{-7}	2.09×10^{-7}
5	100	750	2.00	6.00	3.13×10^{-7}	3.13×10^{-7}
6	100	1000	2.00	6.00	4.17×10^{-7}	4.17×10^{-7}

Table 3: Force between equal masses while varying $m_1 = m_2$.

Trial	m_1 **(kg)**	m_2 **(kg)**	x_1 **(m)**	x_2 **(m)**	$F_{1\to2}$ **(N)**	$F_{2\to1}$ **(N)**
1	50	50	2.00	6.00	1.04×10^{-8}	1.04×10^{-8}
2	100	100	2.00	6.00	4.17×10^{-8}	4.17×10^{-8}
3	250	250	2.00	6.00	2.61×10^{-7}	2.61×10^{-7}
4	500	500	2.00	6.00	1.04×10^{-6}	1.04×10^{-6}
5	750	750	2.00	6.00	2.35×10^{-6}	2.35×10^{-6}
6	1000	1000	2.00	6.00	4.17×10^{-6}	4.17×10^{-6}

Table 4: Force between two masses while varying x_1.

Trial	m_1 **(kg)**	m_2 **(kg)**	x_1 **(m)**	x_2 **(m)**	$F_{1\to2}$ **(N)**	$F_{2\to1}$ **(N)**
1	100	100	0.00	10.00	6.67×10^{-9}	6.67×10^{-9}
2	100	100	2.00	10.00	1.04×10^{-8}	1.04×10^{-8}
3	100	100	4.00	10.00	1.85×10^{-8}	1.85×10^{-8}
4	100	100	6.00	10.00	4.17×10^{-8}	4.17×10^{-8}
5	100	100	8.00	10.00	1.67×10^{-7}	1.67×10^{-7}

Table 5: Force between two masses while varying x_2.

Trial	m_1 **(kg)**	m_2 **(kg)**	x_1 **(m)**	x_2 **(m)**	$F_{1\to2}$ **(N)**	$F_{2\to1}$ **(N)**
1	100	100	0.00	10.00	6.67×10^{-9}	6.67×10^{-9}
2	100	100	0.00	8.00	1.04×10^{-8}	1.04×10^{-8}
3	100	100	0.00	6.00	1.85×10^{-8}	1.85×10^{-8}
4	100	100	0.00	4.00	4.17×10^{-8}	4.17×10^{-8}
5	100	100	0.00	2.00	1.67×10^{-7}	1.67×10^{-7}

Discussion and Analysis

The first major observation that can be made using the raw data is that for all data points (in all 5 tables), $F_{1\to2}$ **(N)** $=$ $F_{2\to1}$ **(N)**. This observation is **Newton's Third Law**, as the force of one mass on another is equal to the other mass on it. Hence, these two forces can be replaced by one force column, denoted as F_g, which is the force of gravitational attraction between the two objects. Also notes that the direction of these two forces are always towards each others, which is why gravitation is known as a force of attraction.

Then, look to Table 1. Observe that there is a relationship between the changing variable (m_1) and the gravitational force. As the other variables are all constant in this experiment, the relationship between m_1 and F_g can be graphed as follows.

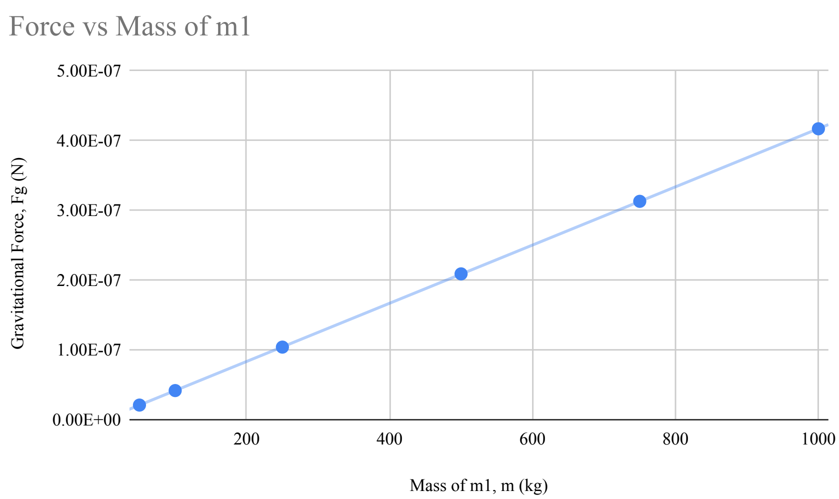

Figure 2: F_g vs m_1 graphed

Observing Figure 2, there is a linear trend between the gravitational force and the mass of object 1. This can be represented by the proportionality:

$$F_g \propto m_1.$$

Hence, the relationship can be described with the below equation, where k_1 is simply a constant:

$$F_g = k_1 \times m_1 \tag{2}$$

Applying the same logic to Table 2 yields the following relationship between F_g and m_2

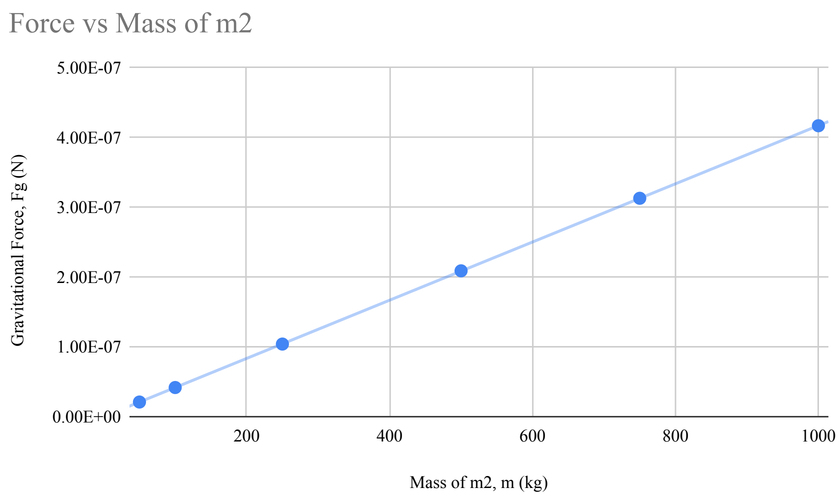

Figure 3: F_g vs m_2 graphed

Note that again, there is a linear trend, implying

$$F_g \propto m_2.$$

Hence, the relationship can be described with the below equation, where k_2 is simply another constant:

$$F_g = k_2 \times m_2. \tag{3}$$

Looking at equation 2 and 3, they can be safely combined into one equation, where there is yet another constant of proportionality k_3:

$$F_g = k_3(m_1 \times m_2). \tag{4}$$

Note, that when m_1 and m_2 are the same, equation 4 simplifies to

$$F_g = k_3 \times m^2.$$

Looking at Table 3, this derived relationship can be verified by graphing as follows:

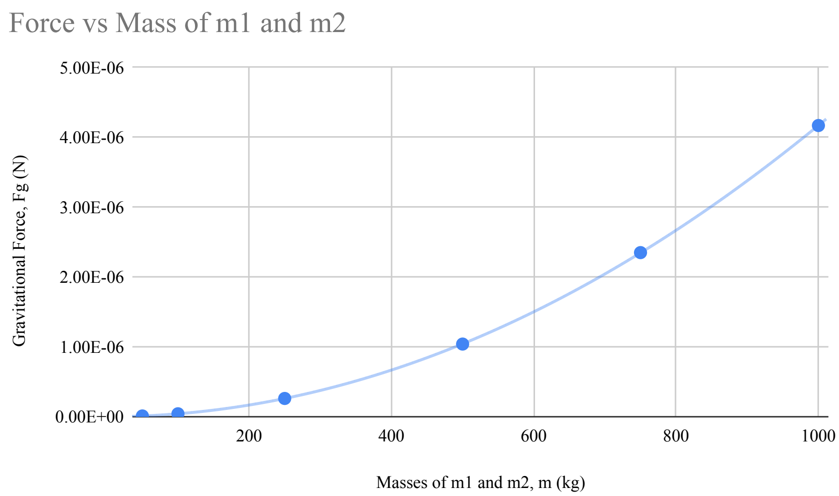

Figure 4: F_g vs mass of $m_1 = m_2$

As suggested by equation 4, there is a proportional quadratic relationship between the masses of the objects and the resulting gravitational force (see Figure 4). Therefore, equation 4 is validated by the simulation.

Moving onto Tables 4 and 5, the output columns for the gravitational force appear identical. Upon further inspection, the gravitational force seems to exhibit a dependency on the distance between the masses. A processed data table can be made combining Tables 4 and 5, showcasing the relationship between F_g and $r := |x_1 - x_2|$.

Table 6: Gravitational force versus distance between two masses.

Distance r (m)	**Gravitational Force F_g (N)**
2.00	1.67×10^{-7}
4.00	4.17×10^{-8}
6.00	1.85×10^{-8}
8.00	1.04×10^{-8}
10.00	6.67×10^{-9}

This new relationship is graphed in the figure below:

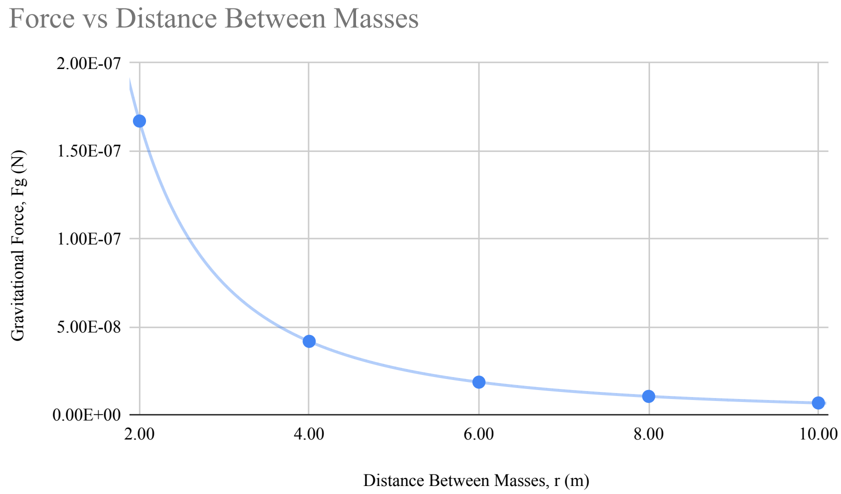

Figure 5: F_g vs $r = x_2 - x_1$

The shown trendline suggests a proportional fit to $\frac{1}{r^2}$ as follows:

$$F_g \propto \frac{1}{r^2},$$

so

$$F_g = k_4 \times \frac{1}{r^2}. \tag{5}$$

When combined with equation 4, NLUG pops out, and the proportionality constant can be denotes as G:

$$F_g = G\frac{m_1 m_2}{r^2} \tag{1}$$

The final step is to solve for the proportionality constant, G. As the relationship has been proven, any data point can be used to solve for this constant. For simplicity, the control data point (used for setup) will be used, where $m_1 = 100$ kg, $m_2 = 100$ kg, $r = x_2 - x_1 = 6\,\text{m} - 2\,\text{m} = 4\,\text{m}$. We have

$$F_g = G\left(\frac{m_1 m_2}{r^2}\right) = G\left(\frac{100\,\text{kg} \times 100\,\text{kg}}{4^2\,\text{m}^2}\right) = 4.17 \times 10^{-8}\,\text{N}$$

$$G = \frac{F_g\, r^2}{m_1 m_2} = \frac{(4.17 \times 10^{-8}\,\text{N})(4\,\text{m})^2}{100\,\text{kg} \times 100\,\text{kg}} = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2$$

.
Ergo, the data from the simulation can be used to derive NLUG and solve for G, the universal gravitational constant:

$$F_g = G\frac{m_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11}\,\text{N}\,\text{m}^2/\text{kg}^2 \tag{6}$$

the force acting on the block can be calculated using Newton's Second Law, which can be used to calculate the frictional coefficients.

Conclusion

Summary

Revisiting the four objectives, the intended goals of this lab were to find the relationship between object masses, distance, and the resulting gravitational force, thereby deriving NLUG and solving for G. The relationship between masses and the gravitational force was found in equation 4:

$$F_g = k_3(m_1 \times m_2). \tag{4}$$

The second objective of finding the relationship between F_g and the distance between the objects is captured in equation 5:

$$F_g = k_4 \times \frac{1}{r^2}. \tag{5}$$

.

The third objective was met by calculating $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$, and the fourth objective was met by deriving NLUG in equation 6:

$$F_g = G\frac{m_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2} \tag{6}$$

The final equation 6 showed the relationship learned in this lab, that gravitation force is proportional to the multiplied masses of the two objects involved divided by the square of the distance between their centers of masses.

Error Analysis

One major source of error within this lab is the assumption that the simulation is entirely functional in simulating gravitational attraction. In an real-world scenario, there are other external forces (friction, air resistance, electromagnetic forces) that can impact the gravitational attraction, and if the simulation doesn't not properly isolate the gravitational force, the final relationship and the calculated gravitational constants can be off. Additionally, the small nature of G means that minor errors in any input variables or rounding errors have a profound impact on the calculated force and gravitational constant. Moreover, the simulation only offers discrete input increments for distance and mass, whereas the scale (particularly for distance) is free to move at any point in the screen. Hence, the distance intervals may be slightly inaccurate as they depend on teh accurate placing of the two objects based on teh scale. Offering continuos distance inputs would offer a slight improvement. Due to the relatively high number of trials conducted, these errors are relatively negligible in finding proportionalities; however, they can significantly impact the calculation of G. Hence, G was calculated using the default values for distance, ensuring that there would be no inaccuracy of the data point. Finally, the simulation uses uniform spheres with a seemingly uniform density. This leads to an unproved generalization that NLUG works for two objects that do not have uniform density (or molecular makeup).

Critiques and Future Applications

The primary critique regarding this lab is that the simulation itself does not render objects as they are in real life, but rather uses NLUG to calculate the output. In other words, NLUG is derived from a simulation where NLUG is already built-in, which nearly invalidates that the simulation as a valid source for data. Using physical objects comes with the additional limitation of human error and external forces; however, they would more accurately showcase the true relationship of NLUG without a biased pre-coded simulation. Additionally, adding more trials would help make the experimental setups more stable, and fixing the errors outlined in the Error Analysis would greatly improve the validity of the experiment itself.

One specific application of NLUG is in calculating satellite orbits. Understanding NLUG allows engineers and physicists to properly determine the force needed to maintain a satellite in stable orbit to ensure proper position for communication. A second application is in planetary motion and astrophysics, where the same gravitational principles are used to predict the trajectories of planets, moons, and comets. Additionally, NLUG can be used to calculate masses of planetary objects, and proper understanding is required for accurate calculations.

This lab can be carried out differently and still illustrate teh same physics concepts. For example, physical masses and a torsion balance can be used (similar to Cavendish) to directly measure the gravitational attraction between small masses in a lab setting. Another method using two pendulums so measure minute changes in motion caused by gravitational attraction. This will allow for a practical visualization of $F_G \propto 1/r^2$. Astronomical observations and using high precision force sensors are also alternatives, although they would not be easily viable for a classroom lab setting.